U. S. NAVAL AIR DEVELOPMENT CENTER

JOHNSVILLE, PENNSYLVANIA

Anti-Submarine Warfare Laboratory

REPORT NO. NADC-AW-6239 31 DEC 1962

THE APPLICATION OF A TRANSFORMATION TECHNIQUE
FOR THE SIMPLIFICATION OF MATHEMATICS
ASSOCIATED WITH THE ASW PROBLEM

PHASE REPORT
WEPTASK NO. RUSD3B000/2021/F004-02-01
Problem No. 023

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In this report a procedure is developed, based on conformal mapping techniques, for efficiently determining the points of intersection of conic sections. In particular, the configurations dealt with here are (1) the determination of the intersections of a hyperbola and ellipse with common foci; (2) the intersection of two ellipses, or equivalently an ellipse and a hyperbola with a common focus.

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S U M M A R Y

INTRODUCTION

In modern airborne data processing, it is imperative that the shortest possible solution for mathematical problems be presented to the computer. It has been observed that the mathematics involved in a "straightforward" approach to some ASW geometrical problems would strain the capabilities of a small scale digital computer. To alleviate this situation, a coordinate transformation methodology has been developed which shows great promise. The method for solving problems pertinent to ASW presented in this report is based on a coordinate transformation technique. A procedure employing these mathematical considerations is developed in appendices A, B, and C.

Two applications are presented as a testimonial for the utility of the transformation technique. Application I is that of determining the coordinates of intersection of an ellipse and a confocal hyperbola. Application II is a similar determination for the intersection of two ellipses having one common focus.

SUMMARY OF RESULTS

1. The prescribed method effects an efficient solution (in so far as digital computation is concerned) for the situations exemplified by applications I and II. The validity of the equations, associated with these applications, has been ascertained by programming them for various combinations of their pertinent parameters.

2. It is implicit in appendix A and explicit in appendices B and C that even though trigonometric and hyperbolic functions are inherent to the development of the proposed technique they need never be contended with directly.

3. When digital computation is to be considered, the transformation technique is preferable to the cosine law technique for the solution of the problem posed by application I (see appendix D).

4. The transformation technique, illustrated in application II is preferable to a competitive method (ellipse-ellipse solution, see appendix D) when digital computation is concerned.

CONCLUSIONS

1. The proposed transformation technique leads to a more efficient method of solution (with regard to digital computation) for the problem posed in application II.

2. Although a cursory comparison of the transformation technique and the cosine law technique (application I) may suggest a slight preference for the latter, on the grounds of general familiarity, the programming analysis of appendix D showed the former to be superior where digital computation is concerned. (It should be noted that a digital computer couldn't care less about the intrinsic nature of the philosophy pertinent to a method of solution.)

3. Appendix D shows that the transformation technique (application II) is preferable to a competitive method where digital computation is concerned.

4. Applications I and II demonstrate the simplifications that may be expected from the utilization of transformation techniques for the solution of simultaneous second-degree equations representing intersecting conic sections which arise in the data processing of many tactical situations. It is expected that a general solution employing such techniques might lead to the optimum means of programming an airborne digital computer to solve problems of this nature.

RECOMMENDATIONS

1. It is recommended that the method illustrated by application II be employed for the digital solution of the pertinent tactical situation.

2. It is recommended that the transformation technique be considered for application to other tactical situations.

3. It is recommended that other transformation techniques be studied for the purpose of reducing, still further, the amount of work to be done by a digital computer.

TABLE OF CONTENTS

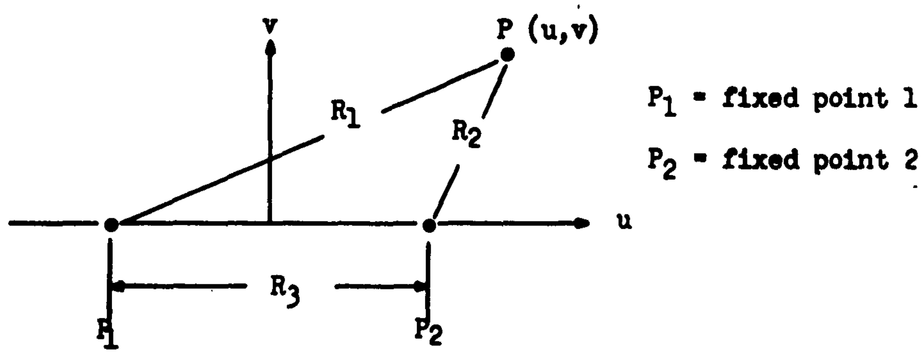
A P P L I C A T I O N S

Application I illustrates a method (based on a transformation technique) for the determination of a target position relative to a coordinate frame defined by two fixed points when the distance between these points and the undirected distances from each point to the target are known. The final equations shown herein are derived in appendix B.

Application II illustrates a method for the determination of the points of intersection of two ellipses with a common focus. (These intersections may also be defined by the intersection of one of the above mentioned ellipses and one branch of a hyperbola.) The equations employed herein are derived in appendix C.

APPLICATION I

Problem: To determine the coordinates (u, v) of a point P when the ranges (R_1 and R_2) to two fixed points are specified. Figure 1 illustrates the pertinent geometry.



Line through P_1 and P_2 defines u axis.
Perpendicular bisector of line joining P_1 and P_2 defines the v axis.

FIGURE 1 - Geometry Pertinent to Application I

Solution Summary

$$u = (A_h)\,(A_e)\,\frac{R_3}{2}$$

$$v = \pm(B_h)\,(B_e)\,\frac{R_3}{2}$$

where $A_e = \dfrac{R_1 + R_2}{R_3}$; $B_e = \pm\sqrt{A^2{}_e - 1}$

$$A_h = \frac{R_1 - R_2}{R_3};\; B_h = \pm\sqrt{1 - A^2{}_h},$$

and the resolution of the ensuing ambiguity is considered to be a separate problem; however, in an application such as DME a prior knowledge of the approximate location of the point P will suffice to yield the desired result.

APPLICATION II

Problem: To determine the location of a target relative to two signal detection points P_1 and P_2.

Given: 1. The coordinates of two signal detection points and that of the signal source.

2. Two numerical values: k_1 and k_2.

where k_i = (i = 1 or 2) difference between the time of arrival of the direct and reflected signals (at signal detection point i) times the velocity of the signal in the applicable medium.

Assumption: Rectilinear signal propogation.

Figure 2 illustrates the pertinent problem geometry.

$$\rho + R_1 - \alpha = k_1$$

$$\rho + R_2 - \beta = k_2$$

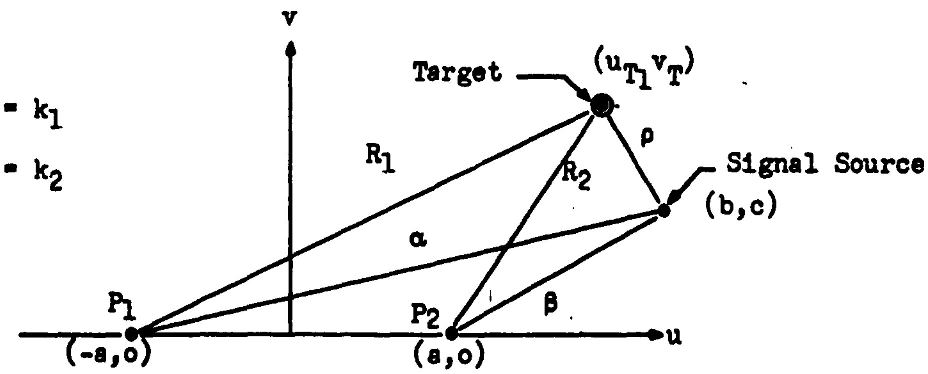


FIGURE 2 - Problem Geometry (Application II)

Given: a k_1
b k_2
c

Find: u_T
v_T

Solution Summary

Computes

$$\alpha = \sqrt{(b + a)^2 + c^2}$$

$$\beta = \sqrt{(b - a)^2 + c^2}$$

$$d_1' = \frac{k_1 + \alpha}{a}$$

$$d_2' = \frac{k_2 + \beta}{a}$$

$$b' = \frac{b}{a}$$

$$c' = \frac{c}{a}$$

$$a_H' = \frac{d_1' - d_2'}{2}$$

$$R = 2\left[b'\ (d_1' - d_2') - d_2' - d_1'\right]$$

$$P = 2c'\left[d_1' - d_2'\right]$$

$$M = (b')^2 + (c')^2 - 1$$

$$Q = d_2'\left[M + (d_1')^2\right] - d_1'\left[M + (d_2')^2\right]$$

$$(b_H')^2 = 1 - (a_H')^2$$

$$A = P^2\ (b_H')^2 - R^2\ (a_H')^2$$

$$B = -2\ QRa_H'$$

$$C = -P^2\ (b_H')^2 - Q^2$$

then, for $A \neq 0$

$$u'_{1,2} = a'_H \left[\frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \right]^*$$

$$\begin{cases} u_{1,2} = a\,u'_{1,2} \\ v_{1,2} = a \left[\dfrac{-Ru'_{1,2} - Q}{P} \right] \end{cases}$$

If $A = 0$

then

$$u_{1,2} = 0$$

and

$$v_{1,2} = \frac{-H \pm \sqrt{H^2 - 4GZ}}{2G}$$

where

$$G = 4\,(c - d_1^2)$$

$$d_1 = ad'_1 \text{ or } K_1 + \alpha$$

$$M = b^2 + c^2 - a^2 - d_1^2$$

$$H = -4\,c\,M$$

$$Z = M^2 - 4d_1^2a^2$$

<u>Result:</u>

Ambiguous solution 1: u_1, v_1

Ambiguous solution 2: u_2, v_2

Where the resolution of the ensuing ambiguity is considered to be a separate problem.

* The subscript 1 implies the use of the (+) sign.
The subscript 2 implies the use of the (-) sign.

APPENDIX A

THE COORDINATE TRANSFORMATION TECHNIQUE

TABLE OF CONTENTS

THE COORDINATE TRANSFORMATION TECHNIQUE

This appendix presents the use of a transformation technique to effect a more efficient solution for a particular problem. The first part discusses transformations in general and develops those particular transformation equations which are employed throughout this report; the latter part discusses the transformation technique which is employed for the determination of the points of intersection of two hyperbolas. (Since this problem is of a complex nature and presents no foreseeable application, the reader may omit it without loss of continuity.)

The solution of many problems becomes very much simpler when the formulas are expressed in terms of a suitable coordinate system. Many tactical situations involve the solution of simultaneous second-degree conic equations such as hyperbolas. The object of this report is to investigate the feasibility of obtaining a more convenient coordinate transformation for these systems, i.e., reduce the problem to the simultaneous solution of linear equations and make lighter the task of an airborne digital computer.

MATHEMATICAL BACKGROUND (COORDINATE TRANSFORMATIONS)

Consider two planes, one in which a point is specified by the numbers x, y and the other in which a point is specified by the numbers u, v. A transformation sets up a correspondence between points in the x, y plane lying in a certain domain and points in a certain domain in the u,v plane. This correspondence may be specified by the transformation equations:

$$u = f(x, y), \quad v = g(x, y)$$

where the functions f and g are required to be single valued so that the point (u, v) is uniquely determined when a point (x, y) is given. It is convenient to call one of the planes, say the x, y plane, the original plane, and the u, v plane the image plane, and say that the point (u, v) is the image of the point (x, y). If the functions f and g are single valued, there is only one image point for every point in the original plane. This does not preclude the possibility that the point (u, v) may not be the image of several points in the x, y plane. If, however, the point (u, v) is the image of only one point in the original plane, the transformation is said to be reversible. The condition for this is that the functions f and g possess unique inverses. The condition for this is simply that the Jacobian of the transformation not vanish over the domain under consideration. Thus:

$$J\left(\frac{u,\ v}{x,\ y}\right) \equiv \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} \neq 0.$$

TECHNIQUE

A procedure is developed, based on conformal mapping procedures, for rapidly determining the point of intersection of a pair of hyperbolas. The method to be described is the geometrical analogue of operational calculus techniques such as the Laplace transformation where differential equations in time are transformed into algebraic equations in terms of a secondary variable. This simpler problem is solved in terms of the secondary variable and then transformed back into the original independent variable to obtain the solution of the original differential equation.

The prescribed procedure calls for the mapping of conic sections in the u, v plane into straight lines in the x, y plane, determining the intersecting point of these straight lines in the x, y plane and then mapping this point back into the u, v plane for the desired solution.

DEVELOPMENT OF EQUATIONS

The transformation $W = \sin Z$ will be employed for this particular study.

$$\begin{aligned} W = u + iv &= \sin Z \\ &= \sin (x + iy) \\ &= \sin x \cos iy + \cos x \sin iy \\ &= \sin x \cosh y + \cos x\ [i \sinh y] \\ &= \sin x \cosh y + i \cos x \sinh y \end{aligned}$$

Therefore, the transformation $W = \sin Z$ can be written

$$u = \sin x \cosh y \tag{A-1}$$

$$v = \cos x \sinh y \tag{A-2}$$

and the line $x = k =$ constant, where $-\frac{\pi}{2} < k < \frac{\pi}{2}$, maps into the curve whose parametric equations are

$$u = \sin k \cosh y \tag{A-3}$$

$$v = \cos k \sinh y \tag{A-4}$$

As an example, if $k = 30° = 0.52360$ radians then

$$u = 0.50000 \cosh y = f_1(y)$$

$$v = 0.86603 \sinh y = f_2(y)$$

Table A-I lists values of u and v for the various values of y in these equations.

TABLE A-I

$u = f_1(y),\ v = f_2(y)$

y	cosh y	u = 0.50000 cosh y	sinh y	v = 0.86603 sinh y
0.0	1.00000	0.50000	0.00000	0.00000
0.2	1.00201	0.51004	0.20134	0.17593
0.4	1.08107	0.54054	0.41075	0.35572
0.5	1.12763	0.56382	0.52110	0.45130
0.7	1.25517	0.62759	0.75858	0.65695
0.9	1.43309	0.71655	1.02652	0.88900
1.1	1.66852	0.83426	1.33565	1.15672
1.3	1.97091	0.98546	1.69838	1.47085

Using the identities

$$\sinh(-y) = -\sinh y$$

$$\cosh(-y) = \cosh y$$

It can be seen that

$$u = 0.50000 \cosh y$$

$$v = -0.86603 \sinh y$$

for negative value of y; therefore, the curve defined by the parametric equations (A-3) and (A-4) will be symmetric with respect to the u axis.

The curve for $x = +30°$ is shown in figure A-1.

Likewise, if $k = -30° = -0.52360$ radians then

$$u = \sin(-30) \cosh y$$

$$v = \cos(-30) \sinh y$$

and using the identities

$\sin(-x) = -\sin x$

$\cos(-x) = \cos x$

then,

$u = -\sin 30^\circ \cosh y$

$v = \cos 30^\circ \sinh y.$

Table A-I can be employed to compute this curve which is also shown in figure A-1.

To demonstrate mathematically that parametric equations (A-3) and (A-4) represent a hyperbola, square (A-3) and (A-4) respectively to obtain

$$u^2 = \sin^2 k \cosh^2 y \tag{A-5}$$

$$v^2 = \cos^2 k \sinh^2 y. \tag{A-6}$$

Substitute the identity

$$\cosh^2 y = 1 + \sinh^2 y \tag{A-7}$$

into equation (A-5) to obtain

$$u^2 = \sin^2 k \,(1 + \sinh^2 y). \tag{A-8}$$

Now, substitute the expression for $\sinh^2 y$ from equation (A-6) into equation (A-8) to obtain

$$u^2 = \sin^2 k \left(1 + \frac{v^2}{\cos^2 k}\right) \tag{A-9}$$

which reduces to the standard form of the hyperbolic equation as follows:

$$\frac{u^2}{\sin^2 k} - \frac{v^2}{\cos^2 k} = 1 = \frac{u^2}{a^2} - \frac{v^2}{b^2} \tag{A-10}$$

letting $\sin k = a$
$\cos k = b.$

For this transformation it has been demonstrated that the following conditions apply:

If $k > 0$, the vertical line in the xy plane maps into the right-hand half of a hyperbola in the uv plane;

If $k < 0$, the vertical line in the xy plane maps into the left-hand half of the hyperbola in the uv plane;

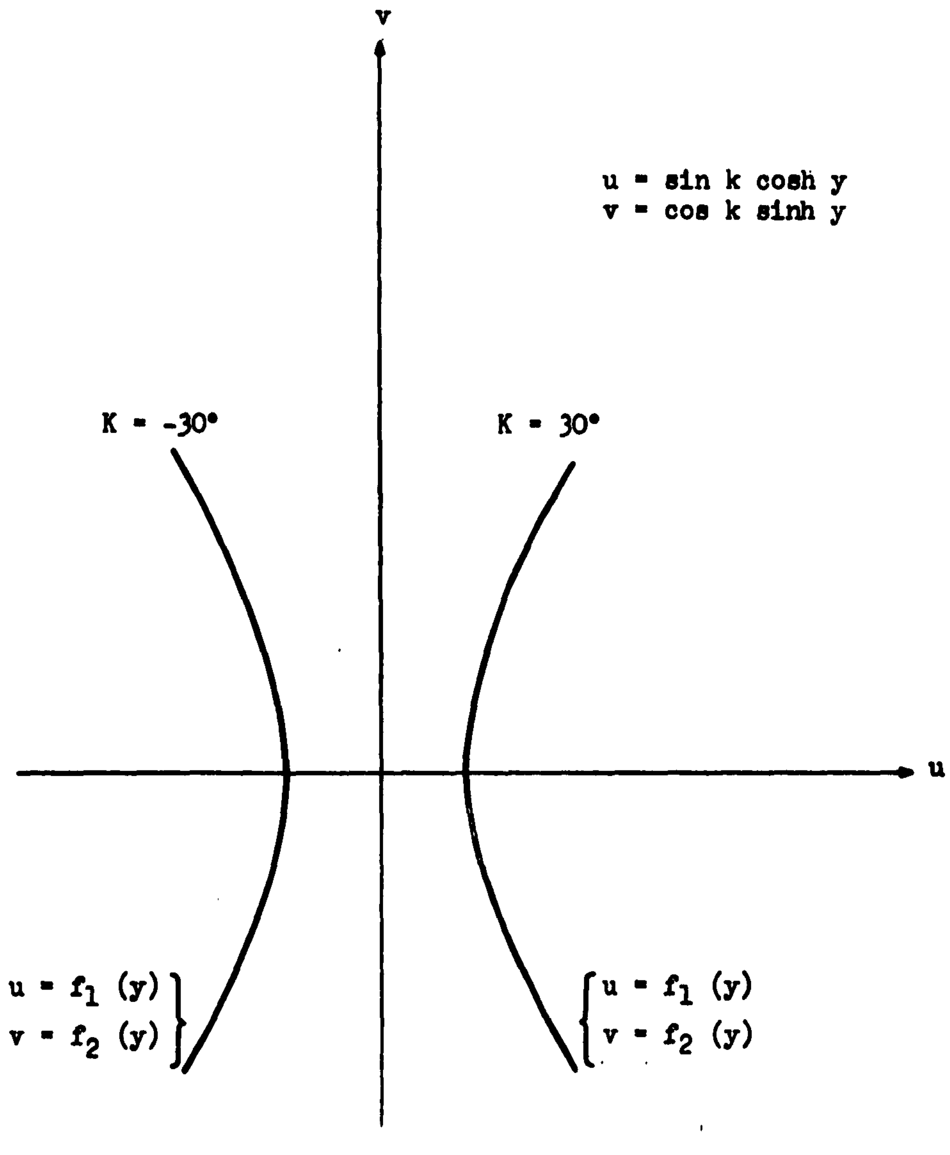


FIGURE A-1 - Plot of Parametric Equations

The mapping is one to one. The points $w = \pm 1$ are the foci of the hyperbola.

From the above it can be seen that vertical lines in the xy plane map into hyperbolas in the uv plane under the transformation $w = \sin Z$.

Conversely, hyperbolas in the uv plane map into vertical lines in the xy plane, since, if w is a function of Z, it follows inversely that Z is a function of w.

EXAMPLE PROBLEM - INTERSECTING HYPERBOLAS

An example of the use of the $w = \sin Z$ transformation in solving problems involving intersecting hyperbolas follows:

The problem which will be solved here is to determine the point of intersection of hyperbola branches No. 1 and No. 2 as illustrated in figure A-2.

As can be seen from figure A-2, hyperbola branch No. 1 may be most easily defined relative to the uv coordinate frame with points f and f' as foci, and similarly hyperbola branch No. 2 may most easily be defined relative to the u'v' coordinate system with points f and f'' as foci.

Also, for this problem the u'v' coordinate system has been translated and rotated through an angle of 90 degrees relative to the uv coordinate system.

The problem has also been designed so that the distances

$$|0f| = |0f'| = |0'f| = |0'f''| = 1.$$

<u>Problem Particulars</u>

The branch of hyperbola No. 1 plotted in figure A-3 from data presented in table A-II, with foci (1, 0) and (-1, 0) in the uv plane, has been designed to pass through the point (3, 2). The hyperbola is defined as the locus of a point which moves so that the difference of its undirected distances from two fixed points is a constant, the value of which is 2a. The value of a is shown in figure A-3.

Choosing the axes as shown in figure A-3 and denoting by 2a the absolute value of the difference between the distances f'P and fP it follows that

$$\sqrt{(u - (-c))^2 + v^2} - \sqrt{(u - c)^2 + v^2} = 2a$$

$$\sqrt{3 - (-1)^2 + 2^2} - \sqrt{(3 - 1)^2 + 2^2} = 2a$$

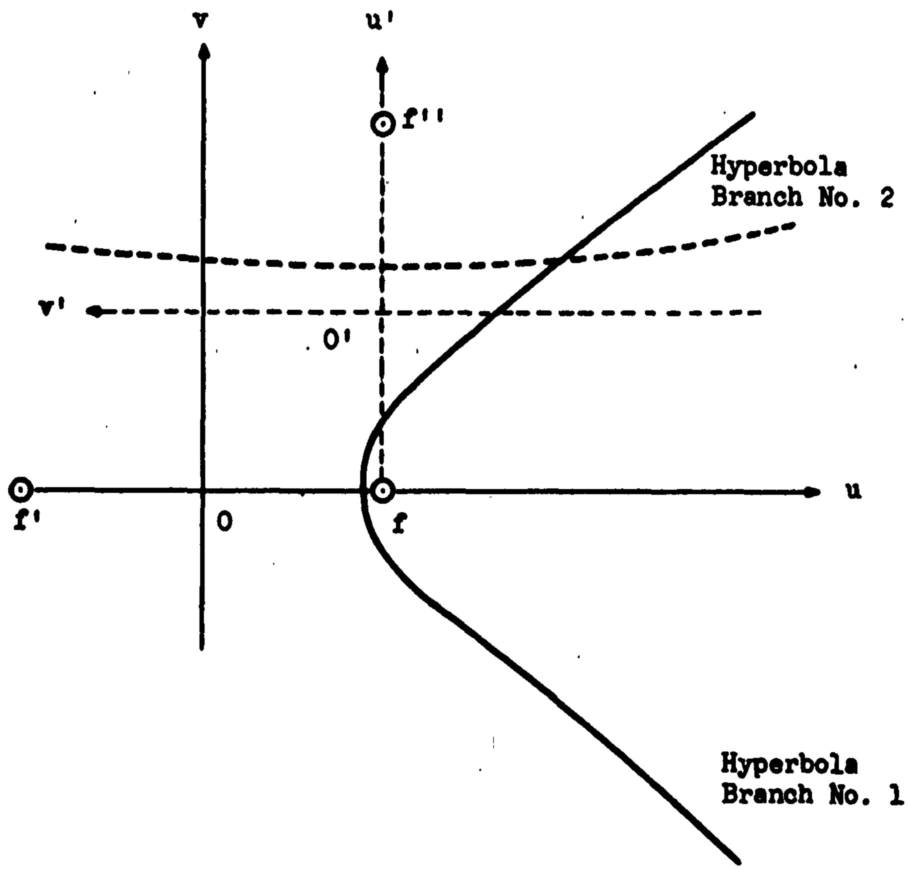


FIGURE A-2 - Sample Problem

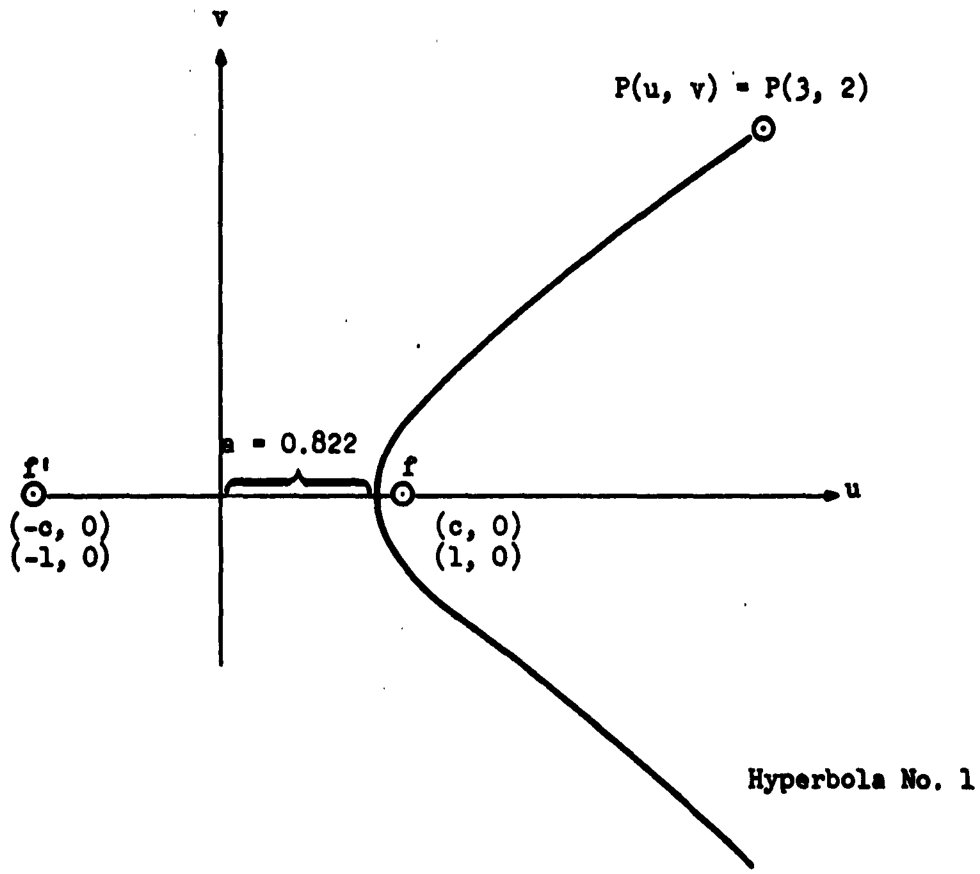


FIGURE A-3 - Set-Up for Hyperbola No. 1

$$\sqrt{16 + 4} - \sqrt{4 + 4} = 2a$$

$$\sqrt{20} - \sqrt{8} = 4.472 - 2.828 = 1.644 = 2a$$

$$a = 0.822.$$

Also, from the relationship

$$c^2 = b^2 + a^2$$

it follows that

$$b^2 = 1 - (0.822)^2 = 1 - 0.676$$

$$b^2 = 0.324,$$

and from

$$\frac{u^2}{a^2} - \frac{v^2}{b^2} = 1$$

the equation

$$v^2 = b^2 \left(\frac{u^2}{a^2} - 1\right) = 0.324 \left(\frac{u^2}{0.676} - 1\right)$$

is derived and employed to develop table A-II from which figure A-3 has been plotted.

TABLE A-II

Points which define the locus of hyperbola No. 1 in the uv plane shown in figure A-2

u	v
0.822	0.000
1.000	±0.395
2.000	±1.263
3.000	±1.999

Similarly, as can be seen from figure A-2, hyperbola No. 2 is defined with the points f and f'' as foci. This hyperbola is set up so that it intersects hyperbola No. 1 at one of the points shown in table A-II such as (2, 1.263) in the uv plane. The most convenient means of obtaining an

equation for this hyperbola would be to establish the u'v' coordinate system as shown in figure A-2, and derive the mathematical expression for hyperbola No. 2 in terms of the u'v' coordinate system.

Also, the point (2, 1.263) in the uv coordinate system is the point (0.263, -1) in the u'v' coordinate system. The set-up for hyperbola No. 2 relative to the u'v' system is shown in figure A-4 as plotted from data developed for table A-III as follows:

Again, from the definition of a hyperbola

$$\sqrt{(u' + c)^2 + v'^2} - \sqrt{(u' - c')^2 + v'} = \pm 2a'$$

$$\sqrt{[0.263 - (-1)]^2 + 1^2} - \sqrt{(1 - 0.263)^2 + 1^2} = 2a'$$

then

$$a' = 0.184$$

and from the relationship

$$c'^2 = b'^2 + a'^2$$

it follows that

$$b'^2 = c'^2 - a'^2$$

$$= 1 - (0.184)^2$$

$$= 0.966.$$

Then, from

$$\frac{u'^2}{a'^2} - \frac{v'^2}{b'^2} = 1$$

the equation

$$v'^2 = b'^2 \left(\frac{u'^2}{a'^2} - 1\right) = 0.966 \left[\frac{u'^2}{(0.184)^2} - 1\right]$$

is employed to develop table A-III.

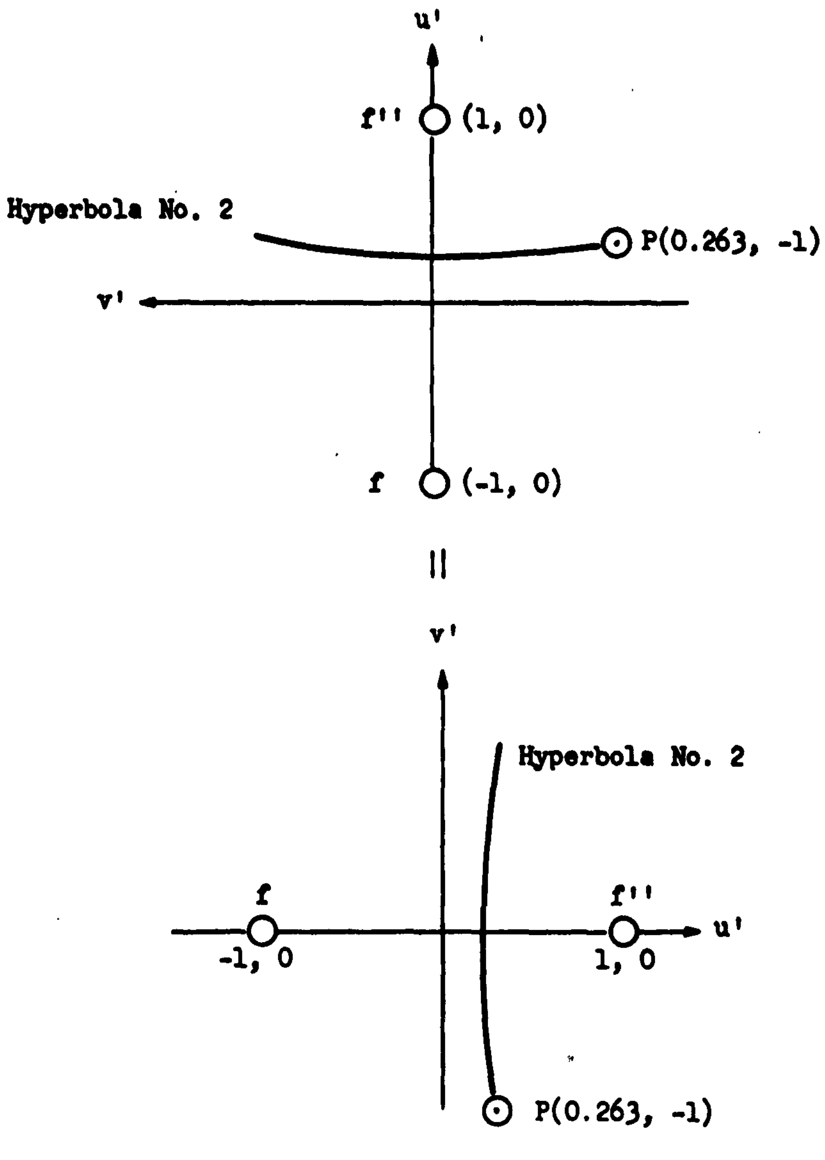


FIGURE A-4 - Set-Up for Hyperbola No. 2

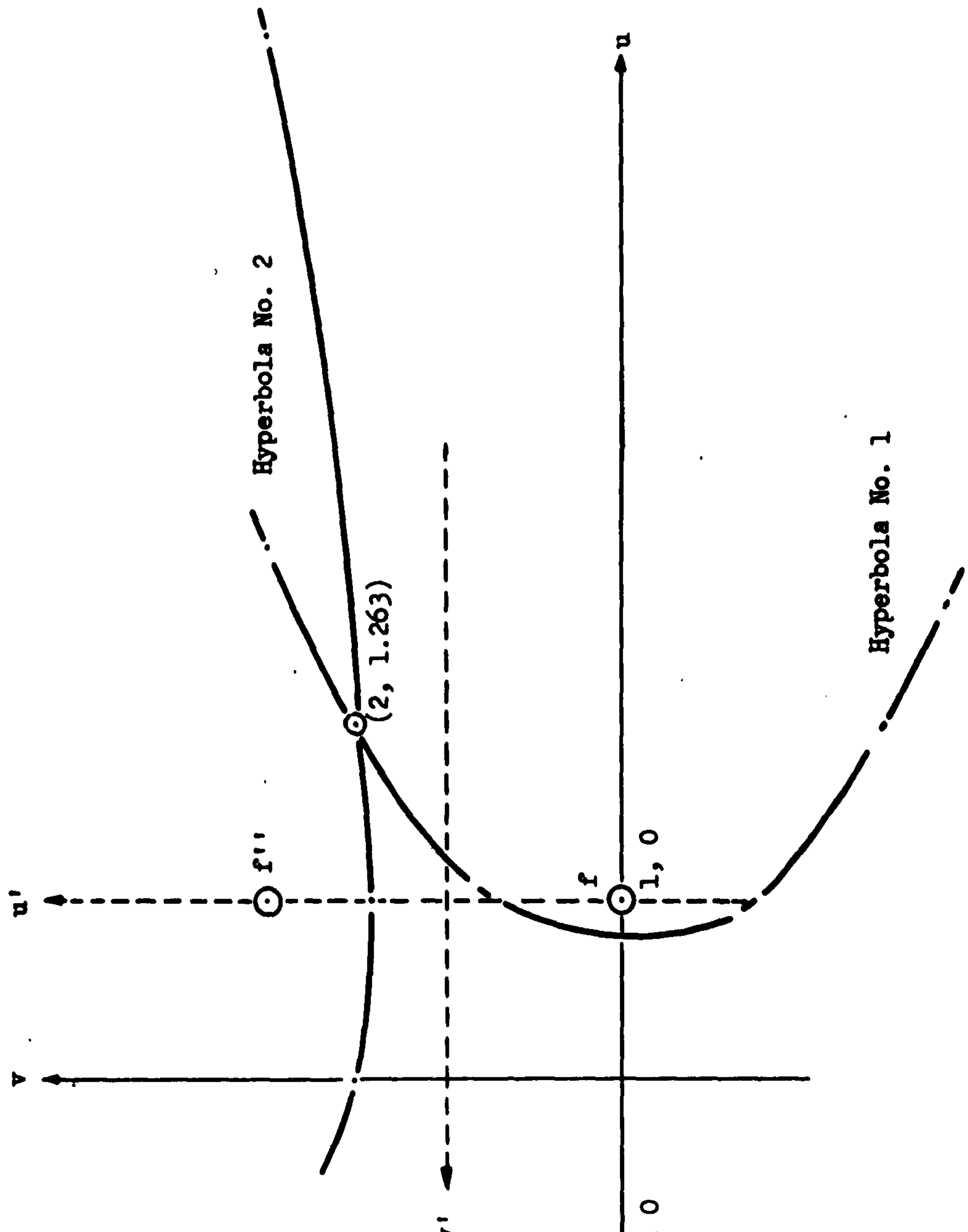


FIGURE A-5 - Problem Illustration

TABLE A-III

Points which define locus of hyperbola No. 2 in u'v' plane

u'	v'
0.184	0.000
0.263	±0.998
1.000	±5.236

To summarize the problem initially illustrated in figure A-2, figure A-5 has been prepared showing the point of intersection of hyperbolas No. 1 and No. 2.

Conventional Method of Solution

The "straightforward" method of solving for the point of intersection of hyperbolas No. 1 and No. 2 (as shown in figure A-5) in the uv plane would be to solve simultaneously:

$$\begin{cases} \dfrac{u^2}{a^2} - \dfrac{v^2}{b^2} = 1 = \dfrac{u^2}{0.676} - \dfrac{v^2}{0.324} & \text{(A-11)} \\[2ex] \dfrac{u'^2}{a'^2} - \dfrac{v'^2}{b'^2} = 1 = \dfrac{u'^2}{0.0340} - \dfrac{v'^2}{0.966} & \text{(A-12)} \end{cases}$$

This technique requires that the equation for hyperbola No. 2 be expressed in terms of the u and v coordinate axes. This is effected, in general, by both a rotation and a translation transformation whereby the general form for the above equations become respectively.

$$A_1 u^2 + C_1 v^2 + F_1 = 0 \tag{A-13}$$

$$A_2 u^2 + B_2 uv + C_2 v^2 + D_2 u + E_2 v + F_2 = 0. \tag{A-14}$$

The prescribed technique for the solution of the above equations involves considerable effort not only in algebraically solving for the desired solution, but also in finding the proper expression for the equations relative to the same coordinate system.

The remainder of this appendix illustrates the proposed conformal transformation technique for the rapid determination of the point of intersection as shown in figure A-5.

DISCUSSION OF THE TRANSFORMATION SOLUTION TECHNIQUE

Hyperbolas No. 1 and No. 2 of figure A-5 will be mapped into straight lines in the xy plane. The point of intersection of these straight lines

in the xy plane will then correspond to the intersection of the hyperbolas in the uv plane, and the inverse transformation of this point back into the uv plane will then yield the desired solution.

Transformation of hyperbola No. 1 from the uv coordinate system to the xy coordinate system is accomplished as follows:

Equation of hyperbola No. 1

$$\frac{u^2}{a^2} - \frac{v^2}{b^2} = 1 = \frac{u^2}{\sin^2 k} - \frac{v^2}{\cos^2 k}$$

by letting a = sin k and b = cos k as previously shown in equation (A-10). Thus, sin k = a = 0.822

$$k = \sin^{-1} 0.822$$

$$k = 55° \; 17'$$

$$k = 0.964 \text{ radians}$$

Figure A-6 is a pictorial exposition utilizing the above computations.

The task of transforming hyperbola No. 2 is not as straightforward as that encountered in the hyperbola No. 1 case. However, while not generally applicable, in this specific case the problem can be attacked without effectively rotating or translating the u'v' coordinate system into the uv coordinate system.

The equation of hyperbola No. 2 relative to the primed system may be written as follows:

$$\frac{u'^2}{a'^2} - \frac{v'^2}{b'^2} = 1 = \frac{u'^2}{\sin^2 k'} - \frac{v'^2}{\cos^2 k'}$$

by letting a' = sin k' and b' = cos k' as previously shown in equation (A-10). Thus,

$$\sin^2 k' = (a^2) = (0.184)^2$$

$$\sin k' = 0.184$$

$$k' = \sin^{-1} 0.184$$

$$k' = 10° \; 38'$$

$$k' = 0.186 \text{ radians.}$$

Since hyperbola No. 1 curving to the right in the u, v plane transformed to a line parallel to the y axis in the x, y plane, it is reasonable to expect that hyperbola No. 2 curving upward in the u', v' plane will transform to a line parallel to the x axis in the x, y plane. The transformed equation would be expected to be as follows relative to the x, y plane

$$y = \text{constant} = \alpha' + k'.$$

Where α' is the y coordinate value of the intersection of the x, y equation for the v' axis relative to the u, v coordinate frame and the line No. 1 in the x, y plane. The transformation equation for the v' line (v = 1) in the uv coordinate system is given by the following equation:

$$v = 1 = \cos x \sinh y$$

or

$$\frac{1}{\cos x} = \sec x = \sinh y.$$

Table A-IV lists values of y for various values of x in the above equation.

TABLE A-IV

x Radians	x Degrees	sec x	y
0.000	0.0	1.0000	0.89
0.262	15	1.0530	0.91
0.523	30	1.1547	0.99
0.785	45	1.4142	1.14
1.047	60	2.0000	1.45
1.309	75	3.8637	2.06
1.571	90		

Table A-IV is employed to plot the above equation in figure A-7 which is an illustration of how both hyperbolas No. 1 and No. 2 map onto the xy plane.

As shown in figure A-7, α' may be interpreted as the transformation of α from the uv coordinate system into the xy coordinate system.

The point of intersection of lines No. 1 and No. 2 in the xy plane uniquely corresponds to the point of intersection of hyperbolas No. 1 and No. 2 in the uv plane.

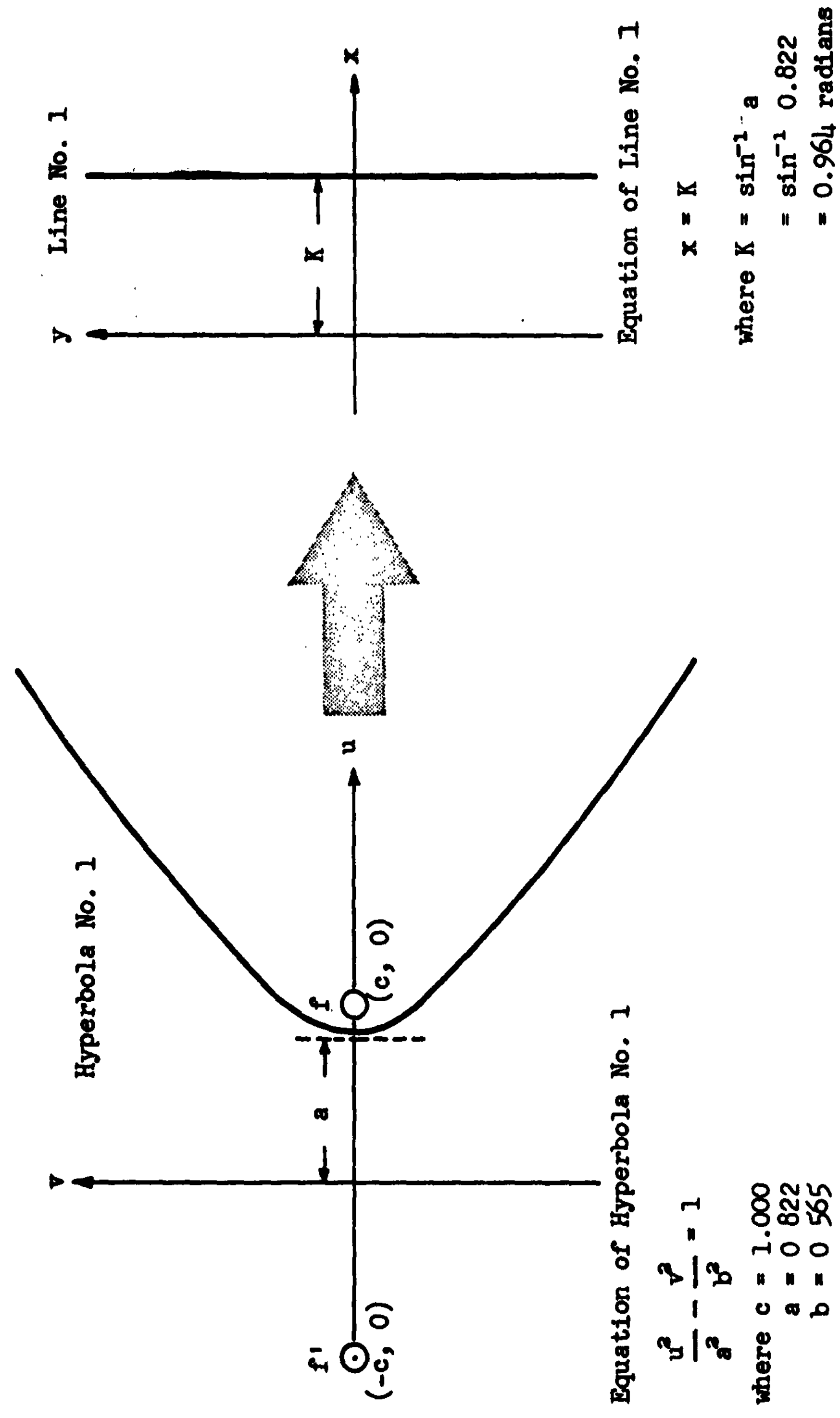


FIGURE A-6 - Hyperbola No. 1 Maps into Line No. 1

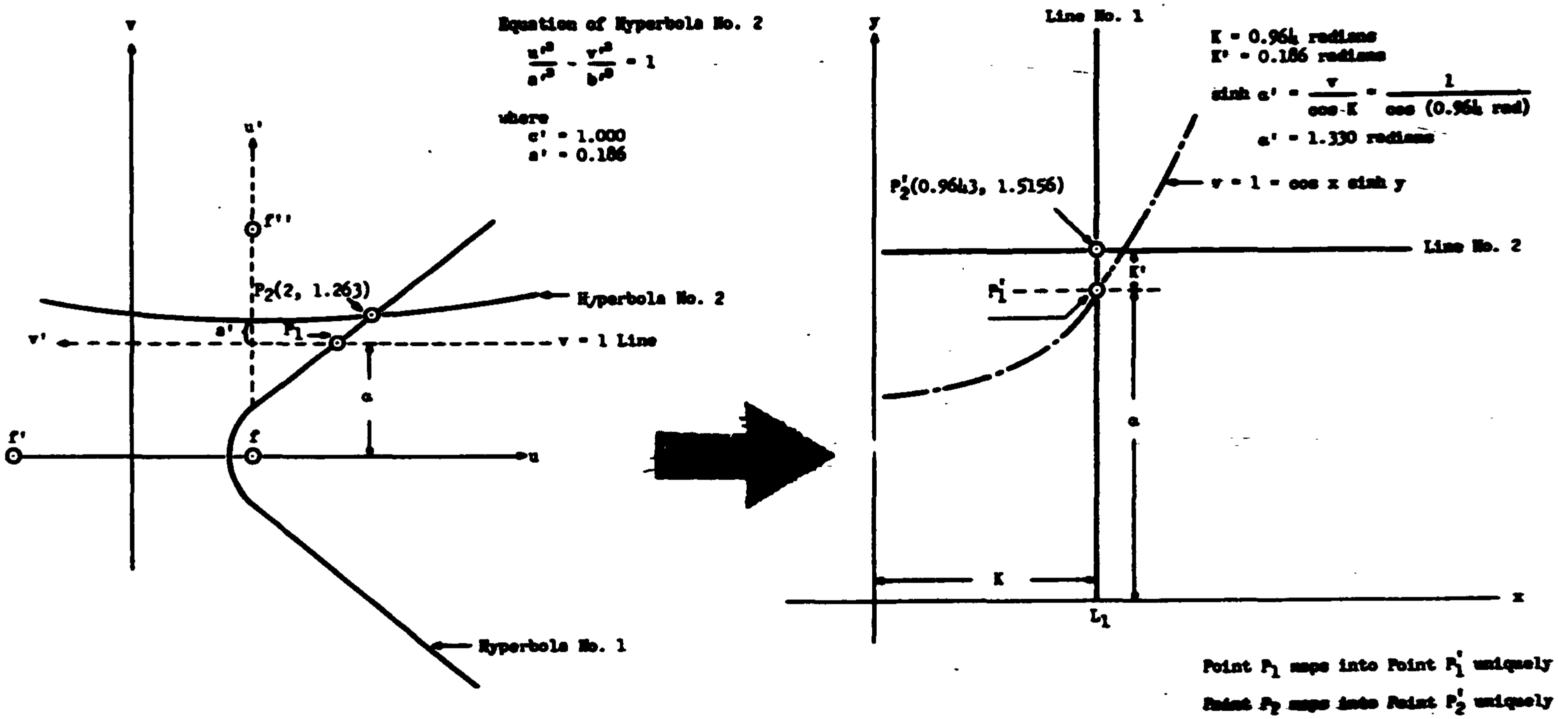


FIGURE A-7 - Mapping of Hyperbolas No. 1 and No. 2 from the uv Plane into the Lines No. 1 and No. 2 Respectively in the xy Plane

The xy coordinates of the point of intersection in the xy plane are (0.9643, 1.5156), and the transformation equations introduced as equations A-1 and A-2 may be employed to map this point into its image point in the uv plane which in this case is the desired solution.

$$u = \sin x \cosh y \tag{A-1}$$

$$= \sin (0.9643 \text{ rad}) \cosh 1.516$$

$$= (0.822)\ (2.395)$$

$$= 1.969$$

$$v = \cos x \sinh y \tag{A-2}$$

$$= \cos (0.964 \text{ rad}) \sinh (1.516)$$

$$= (0.569)\ (2.177)$$

$$= 1.240$$

Therefore, (1.969, 1.240) is the computed point of intersection of the two hyperbolas in the uv plane.

SUMMARY

In summary, the problem of finding the intersection of the following two hyperbolas illustrated in figure A-5 involves the following steps. Given:

Hyperbola No. 1
(Right hand branch only)

$$\frac{u^2}{(0.822)^2} - \frac{v^2}{(0.570)^2} = 1$$

Hyperbola No. 2
(Upper branch only)

$$\frac{u'^2}{(0.184)^2} - \frac{v'^2}{(0.966)^2} = 1$$

Procedure:

$$x = \sin^{-1} 0.822 = 55° \ 17'$$

$$y = \sin^{-1} (0.184) + \alpha'$$

$$\alpha' = \sinh^{-1} \frac{1}{\cos (0.822)} = 1.330$$

$$y = 0.1856 + 1.33 = 1.516$$

$$u = \sin x \cosh y = (0.822)\ (2.395) = 1.969$$

$$v = \cos x \sinh y = (0.570)\ (2.177) = 1.240$$

APPENDIX B

MATHEMATICS ASSOCIATED WITH DISTANCE MEASURING EQUIPMENT (DME)*

(The first practicable application of the "Transformation Technique" is presented herein**)

TABLE OF CONTENTS

* These results are also applicable to other tactical situations having a similar geometrical configuration.

** Authors Note: Since the simultaneous solution of the equations representing two intersecting circles yields a simpler solution than those discussed herein, this appendix is presented for illustrative purposes only.

MATHEMATICS ASSOCIATED WITH DISTANCE MEASURING EQUIPMENT (DME)

THE NAVIGATIONAL PROBLEM

Precise navigation over the surface of the ocean is difficult to accomplish. The object of radio DME is to improve the navigational accuracy of aircraft in ASW operations by electronically locating the aircraft with respect to sonobuoys modified to operate as electronic marine markers. Radio DME systems are not dependent upon good weather conditions, time of day, the operator's eyes, or the proximity to conventional marine markers.

OBJECTIVE

The objective of this appendix is to determine the simpler of two mathematical expressions for the position of an aircraft relative to two sonobuoys.

SYSTEM GEOMETRY

The position of an aircraft may be uniquely determined relative to a coordinate frame defined by two sonobuoys as follows; a straight line through the sonobuoys may conveniently be taken as the u-axis; a straight line through one of the sonobuoys and perpendicular to this u-axis may be taken as the v-axis as shown in figure B-1.

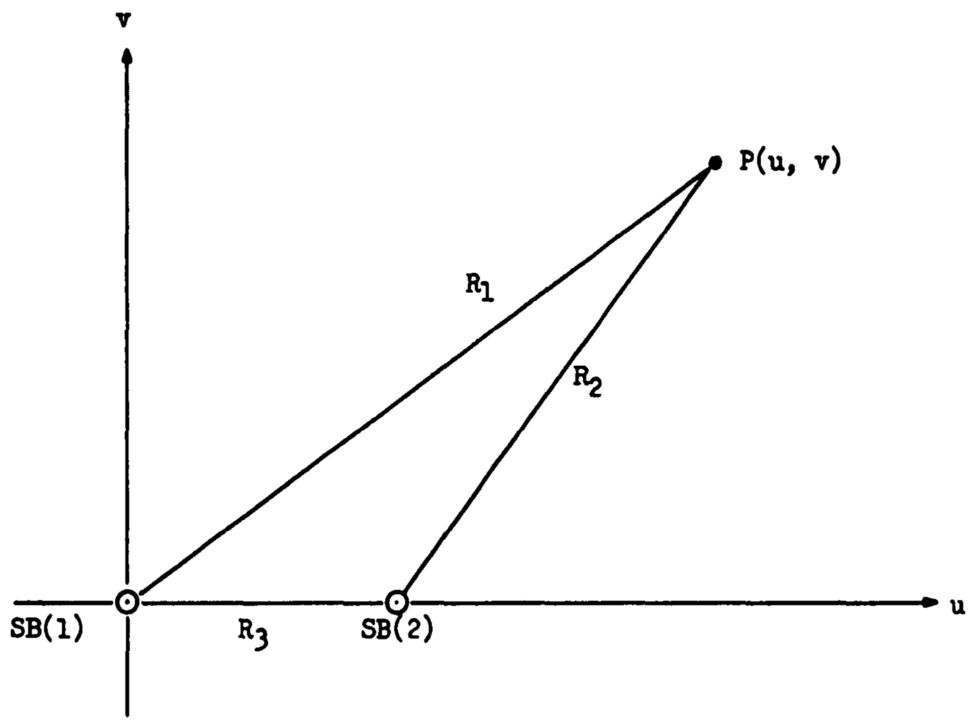
PROBLEM STATEMENT

Two simple mathematical expressions for determining the position of an aircraft, relative to two sonobuoys may be derived from either the "coordinate transformation" or "cosine law" techniques.

Coordinate Transformation Technique

The coordinate transformation technique as described in appendix A may be employed to determine the aircraft position (P) relative to the u-v coordinate frame. This technique is illustrated in figure B-2, where the transformation $w = \sin z$ ($w = u + iv$ and $z = x + iy$) maps the upper half of the ellipse from the u-v plane into the straight line E_{up} in the x-y plane, and the lower half of the ellipse from the u-v plane into the straight line E_{LO} in the xy plane. Also, the right and left halves of the hyperbola in the u-v plane are mapped by this transformation into the straight lines H_R and H_L, respectively, in the x-y plane.

Considering the symmetry of the problem and taking the aircraft to be at position P in the u-v plane, figure B-3 may be constructed to illustrate the application of the transformation technique for the determination of aircraft position.



The symbols shown above are defined as follows:

SB(1) = Position of sonobuoy 1

SB(2) = Position of sonobuoy 2

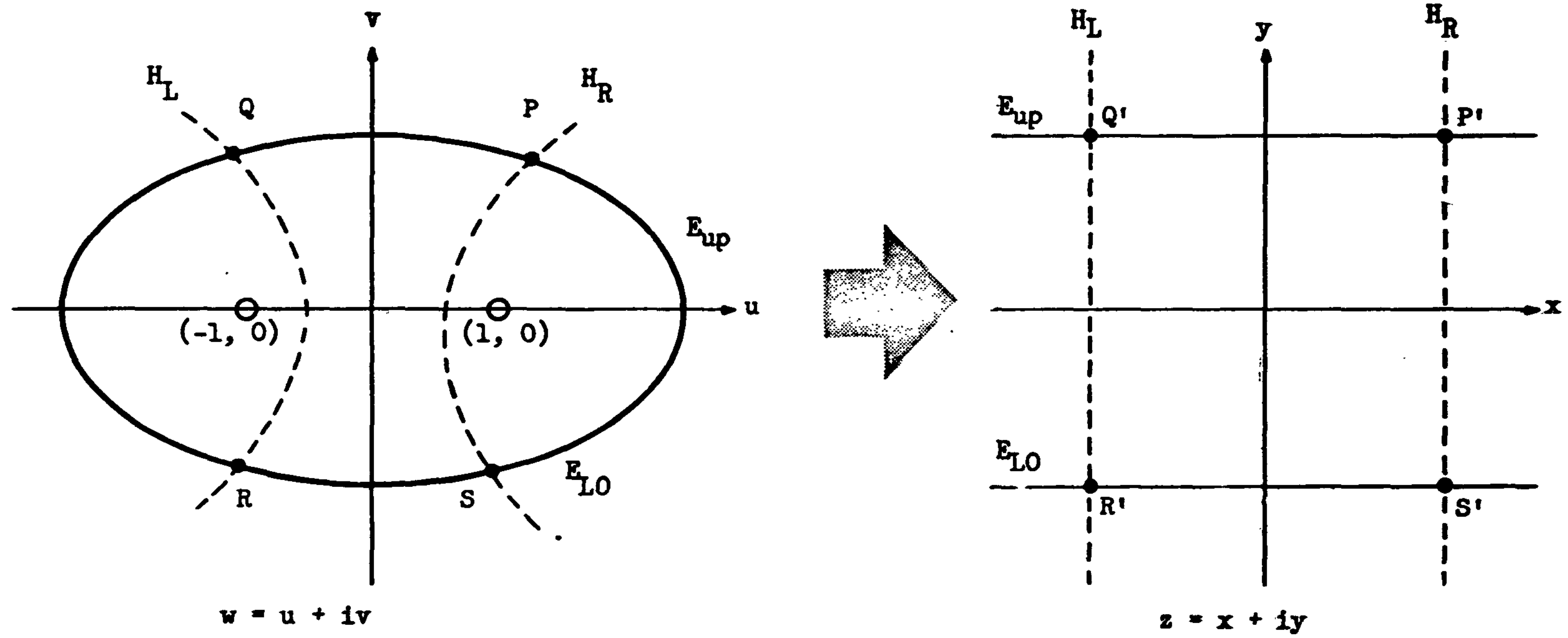
R_1 = Distance between SB(1) and aircraft

R_2 = Distance between SB(2) and aircraft

R_3 = Distance between sonobuoys

P(u, v) = Position of aircraft relative to u, v coordinate frame

FIGURE B-1 - Aircraft Position Relative to a Coordinate Frame Established by Two Sonobuoys



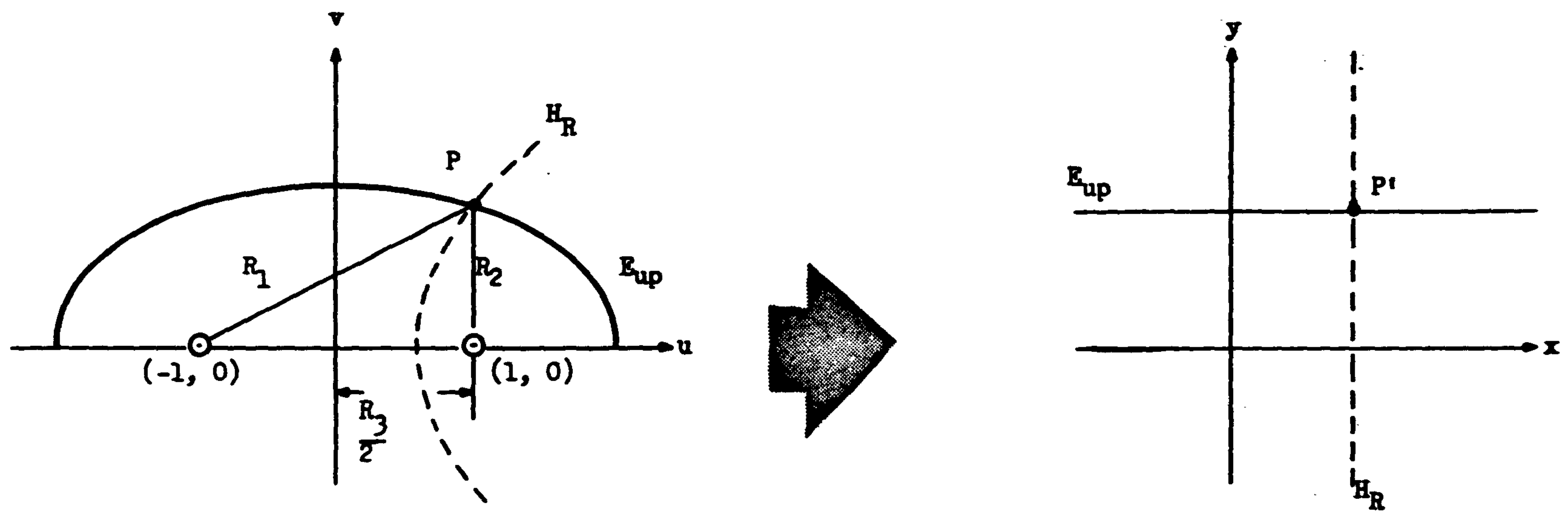
Point P is uniquely mapped into point P'

Point Q is uniquely mapped into point Q'

Point R is uniquely mapped into point R'

Point S is uniquely mapped into point S'

FIGURE B-2 - Coordinate Transformation Technique where the Mapping Function w = sin z



Problem is normalized so that $\frac{R_3}{2} = 1$ plot unit

$u = (A_H)\ (A_E)\ (\frac{R_3}{2})$

$v = (B_H)\ (B_E)\ (\frac{R_3}{2})$

where:

$R_1' = \frac{R_1}{R_3/2};\ R_2' = \frac{R_2}{R_3/2}$

$A_E = \frac{R_1' + R_2'}{2};\ B_E = (A_E^2 - 1)^{1/2}$

$A_H = \frac{R_1' - R_2'}{2};\ B_H = (1 - A_H^2)^{1/2}$

FIGURE B-3 - Equations for Determining Aircraft Position "P" on the u-v Coordinate Frame

In appendix A, it was shown that the coordinates of the aircraft in the u-v coordinate frame could be given as a function of a point (x, y) in the x-y coordinate frame where

$$u = \sin x \cosh y$$

and

$$v = \cos x \sinh y.$$

Now, it is possible to further simplify these expressions so as to avoid the need for evaluating the above trigonometric and hyperbolic functions.

A consideration of the geometry involved, along with the equations given in appendix A will reveal that

$$\sin x = A_H$$

$$\cosh y = A_E$$

$$\cos x = B_H$$

$$\sinh y = B_E$$

where the subscripts E and H refer to the defining ellipse and hyperbola, respectively; that is, 2 A_H is the constant referred to in the classical definition of the hyperbola shown in the u, v plane in figure B-2 which passes through the point P.

Noting that the hyperbola is defined as the locus of a point which moves so that the difference of its undirected distance from two fixed points is a constant, the problem may be arranged as shown in figure B-3 with the scale adjusted so that $R_3/2 = 1$, as required by the theory given in appendix A. In this manner, A_H may be evaluated as

$1/2(R_1' - R_2')$ and B_H may be evaluated as $(1 - A_H^2)^{1/2}$, where

$$R_1' = \frac{R_1}{R_3/2} \text{ and } R_2' = \frac{R_2}{R_3/2}.$$

Also, 2 A_E is the constant referred to in the definition of the ellipse which also passes through the point P.

Noting that the ellipse is defined as the locus of a point which moves so that the sum of its undirected distances from two fixed points is a constant. Again, when the problem is arranged as shown by figure B-3 with the scale adjusted so that $R_3/2 = 1$, A_E may be evaluated as

$$1/2(R_1' + R_2')$$

and B_E may be evaluated as

$$(A_E^2 - 1)^{1/2}$$

where R_1' and R_2' are given above.

Finally, the following equations may be utilized to find the aircraft position "P" in the u-v coordinate frame.

$$u = (A_H)\ (A_E)\ (R_3/2)$$

$$v = (B_H)\ (B_E)\ (R_3/2).$$

The following sample problem (figure B-4) may be utilized to demonstrate the coordinate transformation technique.

Sample Problem

Given:

$$R_1 = 10{,}000.000 \text{ yd}$$

$$R_2 = 8{,}084.736 \text{ yd}$$

$$R_3 = 2{,}000.000 \text{ yd}$$

Solution:

$$A_E = \frac{1}{2}(10 + 8.084736) = \frac{1}{2}\ (18.084736)$$

$$= 9.042368$$

$$A_H = \frac{1}{2}\ (10 - 8.084736) = \frac{1}{2}(1.915264)$$

$$= 0.957632$$

$$u = (A_H)\ (A_E)\ (R_3/2)$$

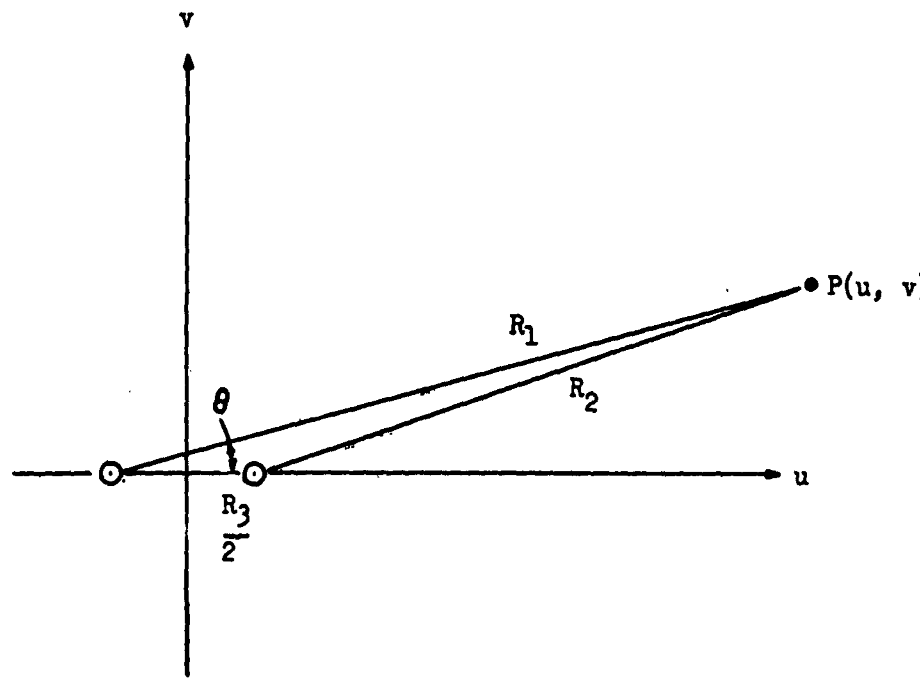
$$= (9.042368)\ (0.957632)\ (1{,}000)$$

$$= 8{,}659.261$$

$$B_E = \sqrt{A_E^2 - 1}$$

$$= \sqrt{(9.04236)^2 - 1} = \sqrt{80.78670096}$$

$$= 8.988142$$



If $R_1 = 10,000$ yd

$R_3/2 = 1,000$ yd

$\theta = 15° \, 00'$

Then, $u = R_1 \cos \theta - R_3/2$

$= 10,000\ (0.96533) - 1,000$

$= 8,659.3$

and $v = R_1 \sin 15°$

$= 10,000\ (0.25882) = 2,588.2.$

Also, using the distance formula

$$R_2 = \sqrt{(u - R_3/2)^2 + v^2} = \sqrt{(7,659.3)^2 + 2,588.2)^2}$$
$$= \sqrt{65.3630 \times 10^6} = 8,084.7.$$

FIGURE B-4 - Sample Problem Arrangement for the Transformation Technique

$$B_H = \sqrt{1 - A_H^2}$$

$$= \sqrt{1 - (0.957632)^2} = \sqrt{1 - 0.9170590047} = \sqrt{0.08294095}$$

$$= 0.287994$$

$$v = (B_E)\ (B_H)\ (R_3/2)$$

$$= (8.988142)\ (0.287994)\ (1{,}000)$$

$$= 2{,}588.530$$

Cosine Law Technique

The cosine law may be stated as follows: "The square of any side of a plane triangle is equal to the sum of the squares of the other two sides diminished by twice the product of those two sides and the cosine of their included angle." Figure B-1 may again be used to illustrate the pertinent geometry of, and the arbitrary coordinate frame for, the solution of the problem. Figure B-5 illustrates the cosine law technique for determining the u-v coordinates of an aircraft relative to two sonobuoys. The sample problem illustrating this technique is shown in figure B-6.

COMPARISON SUMMARY CHART

Given: $R_1 = 10{,}000.0$ yd

$R_2 = 8{,}084.7$ yd

$R_3 = 2{,}000.0$ yd

TECHNIQUES

Transformation

$R_1 = R_1 \div R_3/2 = 10{,}000$

$R_2 = R_2 \div R_3/2 = 8.0847$

$R_3/2 = 1$

$A_E = \frac{1}{2}(10 + 8.0847) = 9.0424$

$A_H = \frac{1}{2}(10 - 8.0847) = 0.95763$

$B_E = (A_E^2 - 1)^{1/2} = (80.7867)^{1/2} = 8.9881$

$B_H = (1 - A_H^2)^{1/2} = \left[1 - (.95763)^2\right]^{1/2} =$

$(0.082941)^{1/2} = 0.28799$

$u = (A_H)(A_E) R_3/2$

$= (9.0424)(0.95763)(1{,}000)$

$= 8{,}659.3$*

$v = (B_E)(B_H)(R_3/2)$

$= (8.9881)(0.28799)(1{,}000)$

$= 2{,}588.5$

Cosine Law

$R_1^2 = (10 \times 10^3)^2 = 100 \times 10^6$

$R_3^2 = (2 \times 10^3)^2 = 4 \times 10^6$

$R_2^2 = (8.0847 \times 10^3)^2 = 65.363 \times 10^6$

$2R_1R_3 = 2(10{,}000)(2{,}000) = 40 \times 10^6$

$R_1^2 + R_3^2 - R_2^2 = 38.637 \times 10^6$

$$\cos\theta = \frac{R_1^2 + R_3^2 - R_2^2}{2R_1R_3} = \frac{38.637 \times 10^6}{40 \times 10^6}$$

$= 0.96593$

$\theta = 15°\ 00'$

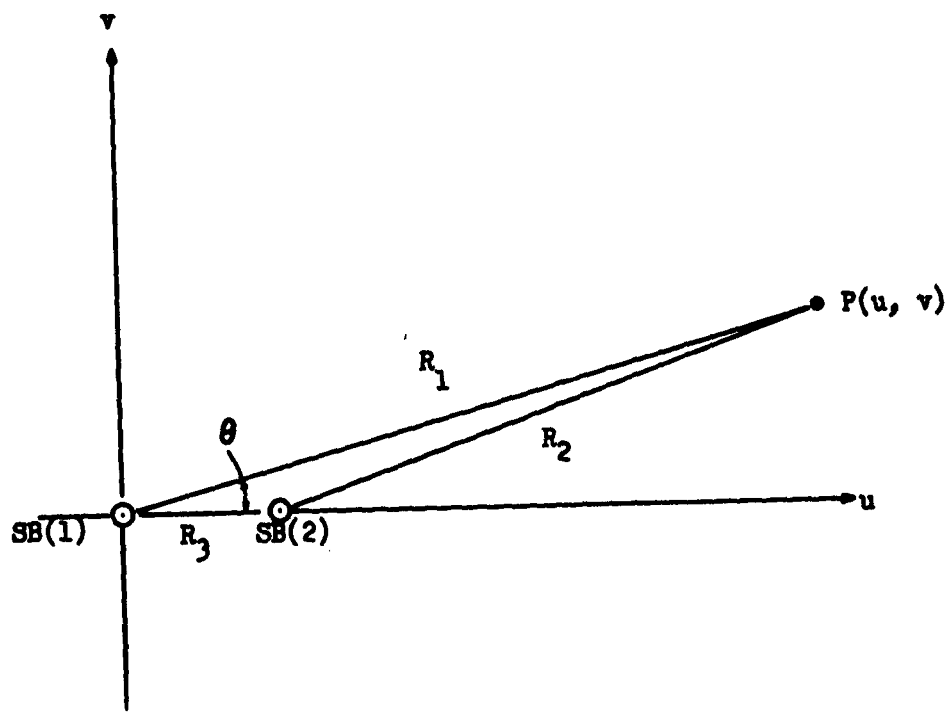
$u = R_1 \cos 15° = 10{,}000\ (0.96593)$

$= 9{,}659.3$

$v = R_1 \sin 15° = 10{,}000\ (0.25882)$

$= 2{,}588.2$

***NOTE:** Recall that the cosine law coordinates have been trasnlated by +1,000 yd along the u axis relative to the transformation technique coordinates.



Law of Cosines:

$$R_2^2 = R_1^2 + R_3^2 - 2R_1R_3 \cos\theta$$

from which

$$\cos\theta = \frac{R_1^2 + R_3^2 - R_2^2}{2R_1R_3}$$

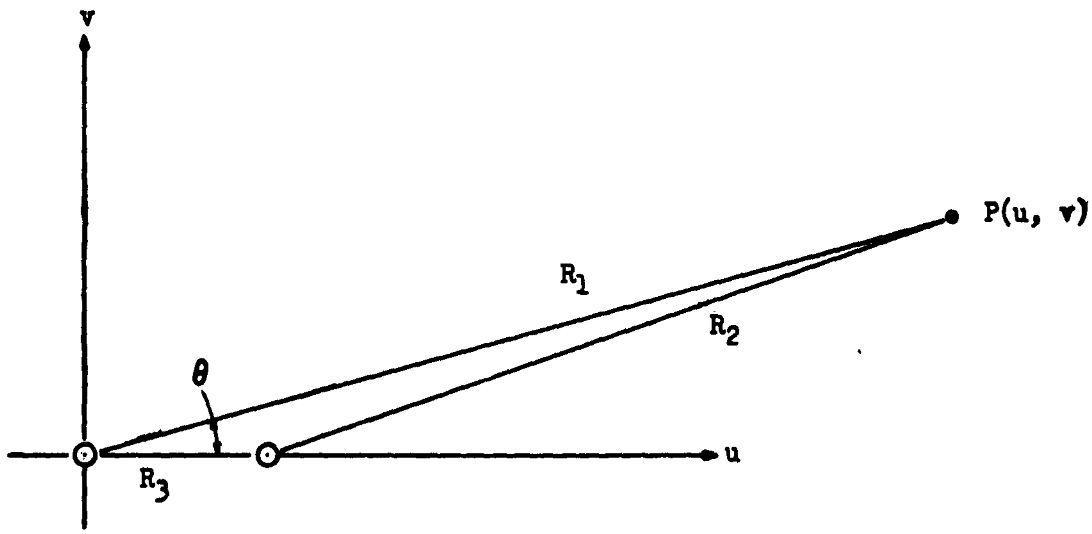
may be obtained.

Then,

$$u = R_1 \cos\theta$$

$$v = R_1 \sin\theta.$$

FIGURE B-5 - Geometry Illustrating the Cosine Law Technique



Given: $R_1 = 10{,}000$ yd

$R_3 = 2{,}000$ yd

$R_2 = 8{,}084.7$ yd

$$\theta = \cos^{-1}\left(\frac{R_1^2 + R_3^2 - R_2^2}{2R_1R_2}\right) = \frac{38.637 \times 10^6}{40 \times 10^6} = 0.06593$$

$$\theta = 15°.$$

Then

$$u = R_1 \cos\theta = 10{,}000\ (0.56593) = 9{,}659.3$$

$$v = R_1 \sin\theta = 10{,}000\ (0.25882) = 2{,}588.2.$$

NOTE: In setting up the problem, R_1 and R_3 were fixed and θ was taken as 15 degrees. u and v were computed and the distance formula was employed to compute R_2.

FIGURE B-6 - Sample Problem Illustrating the Cosine Law Technique

ADDENDUM

By a simple mathematical manipulation it may be shown that A_E and A_H may be computed directly in terms of R_1, R_2, and R_3 by the use of the following formula.

$$A_E = \frac{R_1 + R_2}{R_3}; \quad A_H = \frac{R_1 - R_2}{R_3}.$$

APPENDIX C

MATHEMATICS OF TRANSFORMATION TECHNIQUE APPLIED TO INTERSECTING ELLIPSES

(The equations for the second practicable application of the "Transformation Technique" are developed herein)

TABLE OF CONTENTS

MATHEMATICS OF TRANSFORMATION TECHNIQUE APPLIED TO INTERSECTING ELLIPSES

DISCUSSION

The problem geometry is shown in figure C-1.

$$\rho + R_1 - \alpha = k_1$$

$$\rho + R_2 - \beta = k_2$$

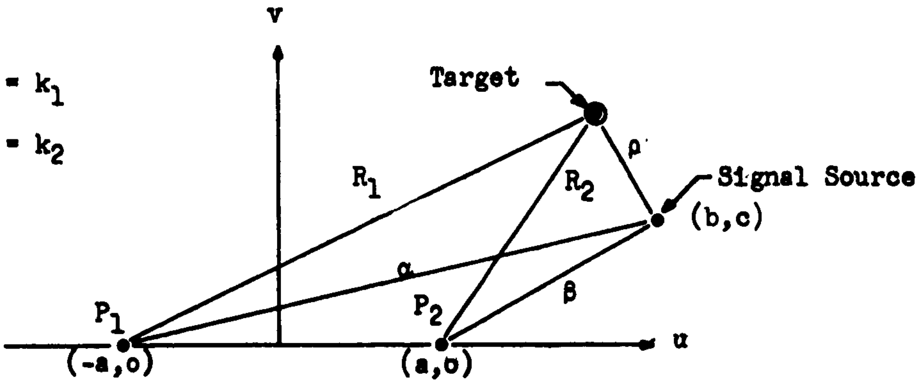


FIGURE C-1 - Problem Geometry

The target location (u, v) may be determined from the simultaneous solution of:

$$\left\{\begin{array}{ll} \sqrt{(u-b)^2+(v-c)^2} + \sqrt{(u+a)^2+v^2} = d_1 & \text{(C-1)} \\ \sqrt{(u-b)^2+(v-c)^2} + \sqrt{(u-a)^2+v^2} = d_2 & \text{(C-2)} \end{array}\right.$$

<u>To solve:</u>

Rewrite equation (C-1) as

$$\sqrt{(u-b)^2+(v-c)^2} - d_1 = -\sqrt{(u+a)^2+v^2}. \qquad \text{(C-3)}$$

Square equation (C-3) to obtain

$$-2bu + b^2 - 2cv + c^2 - 2d_1 \sqrt{(u-b)^2+(v-c)^2}$$
$$+ d_1^2 - 2au - a^2 = 0. \qquad \text{(C-4)}$$

Rewrite equation (C-2) as

$$\sqrt{(u-b)^2+(v-c)^2} - d_2 = -\sqrt{(u-a)^2+v^2}. \qquad \text{(C-5)}$$

Square equation (C-5) to obtain

$$-bu + b^2 - 2cv + c^2 - 2d_2 \sqrt{(u - b)^2 + (v - c)^2}$$
$$+ d_2^2 + 2au - a^2 = 0. \tag{C-6}$$

Arrange terms in equation (C-4) and (C-6) to obtain

$$(-2b - 2a)u - 2cv + b^2 + c^2 + d_1^2 - a^2 - 2d_1 \sqrt{(u - b)^2 + (v - c)^2}$$
$$= 0 \tag{C-7}$$

$$(-2b + 2a)u - 2cv + b^2 + c^2 + d_2^2 - a^2 - 2d_2 \sqrt{(u - b)^2 + (v - c)^2}$$
$$= 0. \tag{C-8}$$

Now, multiply equation (C-7) by d_2, equation (C-8) by d_1, subtract and arrange terms to obtain

$$u\left[d_2 (-2b - 2a) - d_1 (-2b + 2a)\right] + v\left[-2cd_2 - (-2cd_1)\right]$$
$$+ d_2 (b^2 + c^2 + d_1^2 - a^2) - d_1 (b^2 + c^2 + d_2^2 - a^2) = 0. \tag{C-9}$$

Let $R = 2 \left[b(d_1 - d_2) - a (d_2 - d_1)\right]$

$P = 2c \left[d_1 - d_2\right]$

$M = b^2 + c^2 - a^2$

$Q = d_2 (M + d_1^2) - d_1 (M + d_2^2),$

to obtain

$$Ru + Pv + Q = 0. \tag{C-10}$$

Now, to obtain a one-to-one mapping for the hyperbola, the problem must be normalized so that a = 1. Therefore, the geometry of figure C-1 assumes the form of figure C-2.

From figure C-2 it can be seen that

$\rho'' + R_1'$ defines an ellipse - ellipse No. 1

$\rho'' + R_2'$ defines an ellipse - ellipse No. 2

$R_1' - R_2'$ defines a hyperbola

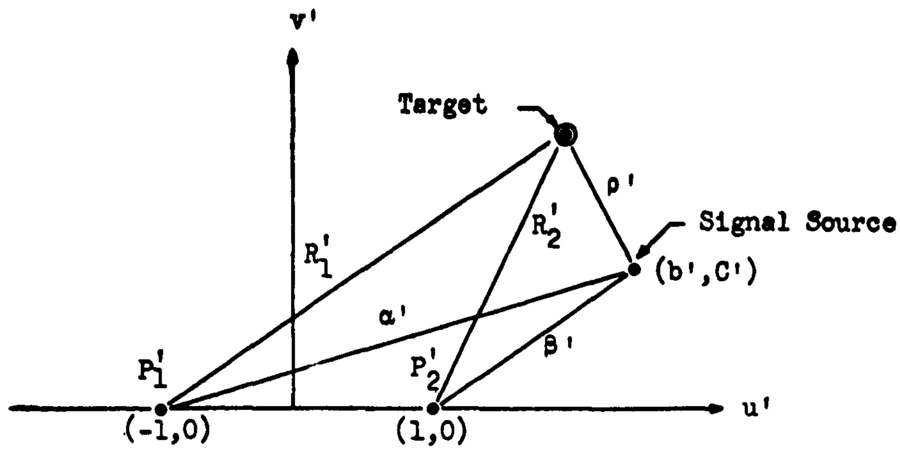


FIGURE C-2 - Normalized Problem Geometry

These conic sections are illustrated in figure C-3

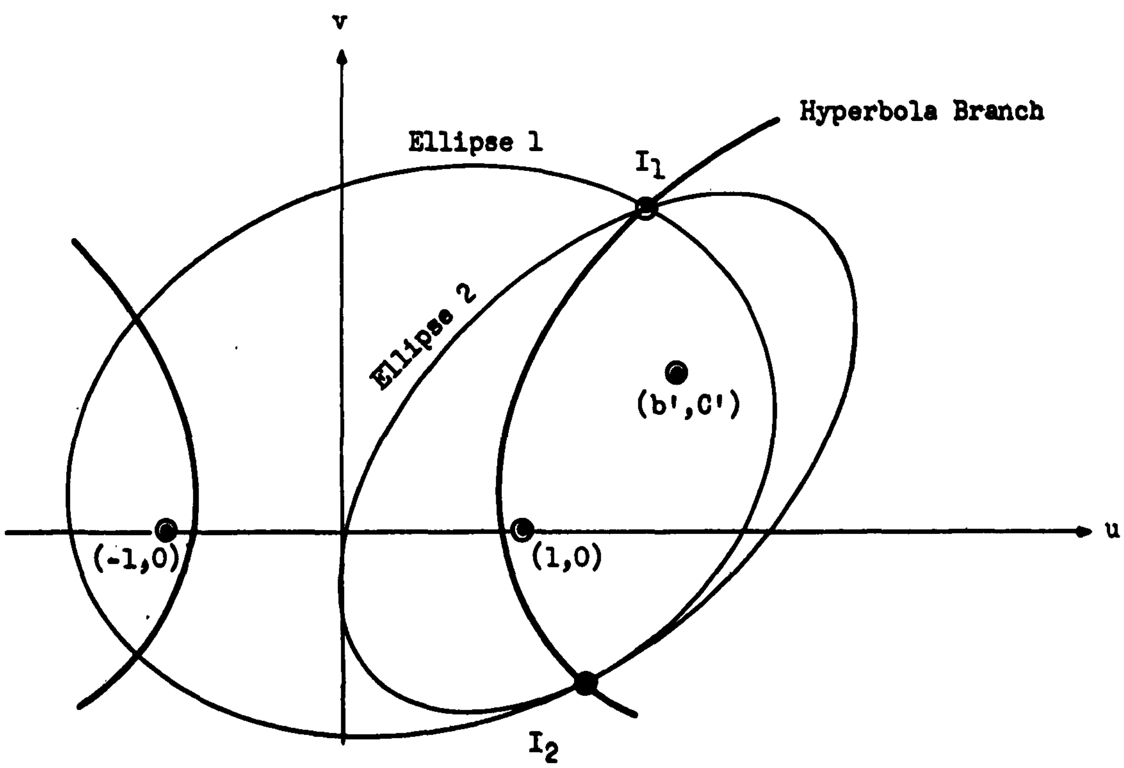


FIGURE C-3 - Conic Sections Pertinent to Problem Solution

The transformation

$$u' = a_H' \cosh y$$

$$v' = b_H' \sinh y$$

where a_H' and b_H' are defined by

$$\frac{(u')^2}{(a_H')^2} - \frac{(v')^2}{(b_H')^2} = 1$$

maps the right hand branch of the hyperbola, shown in figure C-3, into the straight line H_R in the x, y plane as shown in figure C-4.

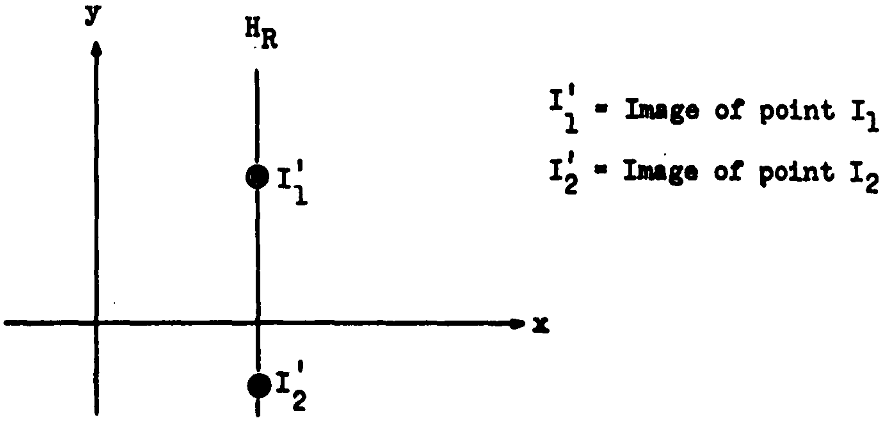


FIGURE C-4 - Map of Right-Hand Branch of the Hyperbola of Figure C-3 into the Straight Line H_R

NOTE: It is not, in general, necessary to know how the ellipses of figure C-3 map into the x-y plane of figure C-4; however, the points of intersection (I_1 and I_2) of ellipses 1 and 2 transform in the same manner as points on the hyperbola. The inverse mapping of points I_1' and I_2' from figure C-4 will uniquely determine points I_1 and I_2 respectively, of figure C-3. These points (I_1 and I_2) are, of course, the solutions that are sought after.

Referring to figure C-2, P, Q, and R become, respectively,

$$P = 2c' \left[d_1' - d_2' \right]$$

$$Q = d_2' \left[M + (d_1')^2 \right] - d_1' \left[M + (d_2')^2 \right]$$

$$R = 2 \left[b' (d_1' - d_2') - d_2' - d_1' \right]$$

where,

$$c' = c/a$$

$$b' = b/a$$

$$d_1' = d_1/a$$

$$d_2' = d_2/a$$

$$M = (b')^2 + (c')^2 - 1$$

and equation (C-10) becomes

$$Ru' + Pv' + Q = 0. \tag{C-11}$$

Now, using the information implicit in figures C-2 and C-3, replace u' and v' in equation (C-11) by a_H' cosh y and b_H' sinh y, respectively, to obtain:

$$R\ a_H' \cosh y + P\ b_H' \sinh y + Q = 0. \tag{C-12}$$

Then,

$$R\ a_H' \cosh y \pm P b_H' \sqrt{\cosh^2 y - 1} + Q = 0$$

and

$$\pm P\ b_H' \sqrt{\cosh^2 y - 1} = -\left[R\ a_H' \cosh y + Q\right]$$

square to obtain

$$P^2 (b_H')^2 \cosh^2 y - P^2 (b_H')^2 = R^2 (a_H')^2 \cosh^2 y + 2QRa_H' \cosh y + Q^2$$

finally,

$$\left[P^2 (b_H')^2 - R^2 (a_H')^2\right] \cosh^2 y - 2\ QRa_H' \cosh y - P^2 (b_H')^2 - Q^2$$
$$= 0. \tag{C-13}$$

Now, let $Z = \cosh y$

$$A = P^2 (b_H')^2 - R^2 (a_H')^2$$

$$B = -2QRa_H'$$

$$C = -P^2 (b_H')^2 - Q^2.$$

To obtain

$$A\, Z^2 + B\, Z + C = 0 \tag{C-14}$$

where

$$Z^*_{1,\,2} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \text{ for } A \neq 0$$

and

$$u'_{1,\,2} = a'_H\, Z_{1,\,2}$$

$$v'_{1,\,2} = \frac{-Ru'_{1,2} - Q}{P}.$$

Finally,

$$\begin{cases} u_{1,\,2} = a\, u'_{1,\,2} \\ v_{1,\,2} = a\, v'_{1,\,2} \end{cases}$$

It may be noted that the above technique does not necessitate the direct computation of either a hyperbolic function or argument thereof per se.

It the target is on the v-axis, a value of $A = 0$ will result and the expression for Z becomes infinite. To avoid this complication, the following alternate must be used.

If $A = 0$

then

$$u_{1,\,2} = 0$$

and

$$v_{1,\,2} = \frac{-H \pm \sqrt{H^2 - 4\, GZ}}{2G}$$

where

$$G = 4\,(c - d_1^2)$$

$$d_1 = ad'_1 \text{ or } K_1 + \alpha$$

$$M = b^2 + c^2 - a^2 - d_1^2$$

$$H = -4\, c\, M$$

$$Z = M^2 - 4d_1^2a^2$$

* The subscript 1 here implies use of the (+) sign.
The subscript 2 here implies use of the (-) sign.

A P P E N D I X   D

PROGRAMMING ANALYSIS
OF
APPLICATIONS I AND II

PROGRAMMING ANALYSIS OF APPLICATIONS I AND II

Submitted by E. A. Bock, Jr

Since there may be more than one acceptable method of solution available (from a strictly mathematical point of view) for a given problem, it was necessary to utilize a methodology for choosing that solution which would be the most efficient when machine computation is to be considered. The criteria used were the following:

1. Precision of the solution (based on a 21 bit data word)
2. Program running time
3. Space required
4. Error analysis

The final program of each method was run on a PB250 computer to verify the results of the methodology.

The final analysis showed that the transformation technique was the most efficient method to use with a computer. Less program analysis was required with the transformation technique than with either of the other two methods tried. Tables D-I and D-II show the results of the evaluation, and clearly indicate that the transformation technique is superior to the other techniques.

TABLE D-I

Type of Solution	Accuracy (Decimal Digits)	Running Time (usec)	Space (21 bit words)
Cosine Law	4	3,240	57
Transformation	5	3,456	56

TABLE D-II

Type of Solution	Accuracy (Decimal Digits)	Running Time (usec)	Space (21 bit words)
Ellipse-Ellipse	3	11,256	192
Transformation	3	9,528	130

DETAILED RESULTS OF THE ANALYSIS OF THE COSINE LAW AND TRANSFORMATION TECHNIQUES

The number of instructions and the running time are essentially the same with both techniques. However, more fundamental analysis was required with the cosine law technique, because an increased amount of scaling was necessary to produce an optimum solution.

The analysis used to determine the maximum number of significant decimal digits, which would be obtained from each solution, revealed that the transformation technique result contained at least one more significant digit; therefore, it would produce more accurate results (with the same input) than the cosine law technique.

DETAILED RESULTS OF THE ANALYSIS OF THE ELLIPSE-ELLIPSE AND TRANSFORMATION TECHNIQUES

The analysis showed that even when the input data was normalized, the competitive technique required a "floating point" routine to produce a solution. Although the use of the floating point routine reduced the analysis required for scaling, it slowed down the running time needed to produce a solution. This increase in time is not reflected in table D-II; this was not the case with the transformation technique.

The running time for the ellipse-ellipse technique was much longer than for the transformation technique. This was due partly to the increased number of instructions needed and partly to an increased number of divide, multiply and square root instructions needed.

With the criteria used, the results of the analysis show that the transformation technique was the most efficient program to use when solving for the points of intersection of conic sections more complex than circles.

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In this report a procedure is developed, based on conformal mapping techniques, for efficiently determining the points of intersection of conic sections. In particular, the configurations dealt with here are (1) the determination of the intersections of a hyperbola and ellipse with common foci; (2) the intersection of two ellipses, or equivalently an ellipse and a hyperbola with a common focus.

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